

## Annex to the paper

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## Solving NLS Equation for a Nonlinear Media using Volterra series

Volterra series expansion is one of the most powerful methods for describing the non-linear systems. We will now obtain the output of the nonlinear system by solving NLS equations with Volterra series. If we consider the first 3 terms of the Volterra series we will have a good approximation of the solution of NLS equation. So  $X_o(\omega, z)$  can be evaluated as:

$$X_o(\omega, z) = X_{o_1}(\omega, z) + X_{o_2}(\omega, z) + X_{o_3}(\omega, z)$$

Where  $X_{o_1}(\omega, z)$ ,  $X_{o_2}(\omega, z)$  and  $X_{o_3}(\omega, z)$  are defined as follows

$$\begin{aligned} X_{o_1}(\omega, z) &= \iint H'_{o_{1,2}}(\omega_1, \omega_2, \omega - \omega_1 + \omega_2, z) \times C(\omega_1)S^*(\omega_2)C(\omega - \omega_1 + \omega_2)d\omega_1 d\omega_2 \\ X_{o_2}(\omega, z) &= \iiint H'_{o_{3,2}}(\omega_1, \omega_2, \omega_3, \omega_4, \omega - \omega_1 + \dots + \omega_4, z) \times C(\omega_1)S^*(\omega_2)S(\omega_3)S^*(\omega_4) \\ &\quad \times C(\omega - \omega_1 + \dots + \omega_4)d\omega_1 \dots d\omega_4 \\ X_{o_3}(\omega, z) &= \iiint H'_{o_{1,4}}(\omega_1, \omega_2, \omega_3, \omega_4, \omega - \omega_1 + \dots + \omega_4, z) \times C(\omega_1)C^*(\omega_2)C(\omega_3)S^*(\omega_4) \\ &\quad \times C(\omega - \omega_1 + \dots + \omega_4)d\omega_1 \dots d\omega_4 \end{aligned}$$

Where  $c(t)$ ,  $s(t)$  are the clock and the received signal in time domain,  $C(\omega)$ ,  $S(\omega)$  are the Fourier transform of the clock and received signal and the Volterra kernels,  $H'_{o_{1,2}}$ ,  $H'_{o_{3,2}}$  and  $H'_{o_{1,4}}$  are calculated as follows

$$\begin{aligned} H'_{o_{1,2}}(\omega_1, \omega_2, \omega - \omega_1 + \omega_2, z) &= 2j\gamma_{osc}e^{G_1(\omega)z}\left(\frac{1 - e^{-2\alpha z}}{2\alpha}\right) \\ H'_{o_{3,2}}(\omega_1, \omega_2, \omega_3, \omega_4, \omega - \omega_1 + \dots + \omega_4, z) &= 2\gamma_{osc}(\gamma_{ss} + 2\gamma_{os})e^{G_1(\omega)z}\left(\frac{1 - e^{-2\alpha z}}{2\alpha}\right)^2 \\ H'_{o_{1,4}}(\omega_1, \omega_2, \omega_3, \omega_4, \omega - \omega_1 + \dots + \omega_4, z) &= 8\gamma_{osc}(\gamma_{sc} + 2\gamma_{oc})e^{G_1(\omega)z}\left(\frac{1 - e^{-2\alpha z}}{2\alpha}\right)^2 \end{aligned}$$

Where  $\alpha$  is the linear attenuation constant,  $\gamma_{ijkl}$  is spatial distribution of the fiber mode. For simplicity of calculation we suppose that the received and the clock signals are Sinc shape. So these signals in time and spectral domain as follow:

$$\begin{aligned}
c(t) &= \sum_{m=1}^k A_c' \cdot \text{Sinc} \left( \frac{t - (C_m \cdot T_b / F)}{T_c} \right) \\
C(\omega) = F\{c(t)\} &= A_c' \cdot T_c \cdot \pi \cdot \text{rect} \left( \frac{\omega \cdot T_c}{2} \right) \cdot \sum_{m=1}^K e^{-2\pi j C_m \cdot (T_b / F) \cdot \omega} \\
s(t) &= b \cdot \sum_{m=1}^k A_d \cdot \text{Sinc} \left( \frac{t - (C_m \cdot T_b / F)}{T_d} \right) \\
S(\omega) = F\{s(t)\} &= b \cdot A_d \cdot T_d \pi \cdot \text{rect} \left( \frac{\omega \cdot T_d}{2} \right) \cdot \sum_{m=1}^k e^{-2\pi j C_m \cdot (T_b / F) \cdot \omega}
\end{aligned}$$

Where  $A_c', A_d$  are amplitudes of the clock and the received signals,  $C_m$  is the code's number,  $k$  is the weight of the code,  $T_d, T_c$  are full width at half maximum (FWHM) of the data pulse and the clock pulse-width (code word),  $T_b$  is duration of bit and  $b$  is bit value can be "0", "1". So we can write the first three output response terms as follow:

$$\begin{aligned}
X_{o_1}(\omega, z) &= \iint 2j\gamma_{osc} e^{G_1(\omega)z} \left( \frac{1 - e^{-2\alpha z}}{2\alpha} \right) \times \{A_c' \cdot T_c \cdot \pi \cdot \text{rect} \left( \frac{\omega_1 \cdot T_c}{2} \right) \cdot \sum_{m=1}^K e^{-2\pi j C_m \cdot (T_b / F) \cdot \omega_1}\} \\
&\times \left\{ b \cdot A_d \cdot T_d \pi \cdot \text{rect} \left( \frac{\omega_2 \cdot T_d}{2} \right) \cdot \sum_{m=1}^k e^{2\pi j C_m \cdot (T_b / F) \cdot \omega_2} \right\} \\
&\times \{A_c' \cdot T_c \cdot \pi \cdot \text{rect} \left( \frac{(\omega - \omega_1 + \omega_2) \cdot T_c}{2} \right) \cdot \sum_{m=1}^K e^{-2\pi j C_m \cdot (T_b / F) \cdot (\omega - \omega_1 + \omega_2)}\} d\omega_1 d\omega_2
\end{aligned}$$

$$\begin{aligned}
X_{o_2}(\omega, z) &= \iint \iint 2\gamma_{osc} (\gamma_{ss} + 2\gamma_{os}) e^{G_1(\omega)z} \left( \frac{1 - e^{-2\alpha z}}{2\alpha} \right)^2 \\
&\times \{A_c' \cdot T_c \cdot \pi \cdot \text{rect} \left( \frac{\omega_1 \cdot T_c}{2} \right) \cdot \sum_{m=1}^K e^{-2\pi j C_m \cdot (T_b / F) \cdot \omega_1}\} \\
&\times \left\{ b \cdot A_d \cdot T_d \pi \cdot \text{rect} \left( \frac{\omega_2 \cdot T_d}{2} \right) \cdot \sum_{m=1}^k e^{2\pi j C_m \cdot (T_b / F) \cdot \omega_2} \right\} \\
&\times \left\{ A_c' \cdot T_c \cdot \pi \cdot \text{rect} \left( \frac{\omega_3 \cdot T_c}{2} \right) \cdot \sum_{m=1}^K e^{-2\pi j C_m \cdot (T_b / F) \cdot \omega_3} \right\} \\
&\times \left\{ b \cdot A_d \cdot T_d \pi \cdot \text{rect} \left( \frac{\omega_4 \cdot T_d}{2} \right) \cdot \sum_{m=1}^k e^{2\pi j C_m \cdot (T_b / F) \cdot \omega_4} \right\}
\end{aligned}$$

$$\times \{A_c'.T_c.\pi.\text{rect}\left(\frac{(\omega - \omega_1 + \dots + \omega_4).T_c}{2}\right).\sum_{m=1}^K e^{-2\pi j C_m.(T_b/F).(\omega - \omega_1 + \dots + \omega_4)}\} \times d\omega_1 \dots d\omega_4$$


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$$X_{o_3}(\omega, z) = \iint \iint 8\gamma_{osc}(\gamma_{sc} + 2\gamma_{oc})e^{G_1(\omega)z}\left(\frac{1 - e^{-2\alpha z}}{2\alpha}\right)^2$$

$$\times \{A_c'.T_c.\pi.\text{rect}\left(\frac{\omega_1.T_c}{2}\right).\sum_{m=1}^K e^{-2\pi j C_m.(T_b/F).\omega_1}\},$$

$$\times \left\{A_c'.T_c.\pi.\text{rect}\left(\frac{\omega_2.T_c}{2}\right).\sum_{m=1}^K e^{2\pi j C_m.(T_b/F).\omega_2}\right\}$$

$$\times \left\{A_c'.T_c.\pi.\text{rect}\left(\frac{\omega_3.T_c}{2}\right).\sum_{m=1}^K e^{-2\pi j C_m.(T_b/F).\omega_3}\right\}$$

$$\times \left\{b.A_d.T_d\pi.\text{rect}\left(\frac{\omega_4.T_d}{2}\right).\sum_{m=1}^k e^{2\pi j C_m.(T_b/F).\omega_4}\right\}$$

$$\times \{A_c'.T_c.\pi.\text{rect}\left(\frac{(\omega - \omega_1 + \dots + \omega_4).T_c}{2}\right).\sum_{m=1}^K e^{-2\pi j C_m.(T_b/F).(\omega - \omega_1 + \dots + \omega_4)}\} \times d\omega_1 \dots d\omega_4$$

By calculating the integration in (), (), () and depending on whether  $T_c$  is larger than  $T_d$  or not, we obtain different responses in different frequency ranges. By defining  $f(x)$  and  $E_1$  as:

$$f(x) = \frac{1}{2\pi j(T_b/F)} e^{-2\pi j(T_b/F).x}$$

$$, E_1 = 2\pi^3 b j \gamma_{osc} e^{G_1(\omega)z} \left(\frac{1 - e^{-2\alpha z}}{2\alpha}\right) A_d A_c'^2 T_d T_c^2$$

Then, for  $T_d > T_c$ ,  $X_{o_1}$  would be:

$$(1) \text{ For } \omega > \frac{2}{T_c} + \frac{1}{T_d} \rightarrow X_{o_1}(\omega, z) = 0$$

$$(2) \text{ For } \frac{1}{T_d} < \omega < \frac{2}{T_c} + \frac{1}{T_d}$$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{1}{(C_l - C_m)} \cdot \frac{2\pi j(T_b/F)}{(C_i + C_m - 2C_l)} f\left((C_l - C_i)\frac{2}{T_c}\right) \cdot f\left((C_i - C_m - 2C_l)\left(\frac{1}{T_d}\right)\right) \cdot f(C_i(\omega))$$

$$(3) \text{ For } \frac{-2}{T_c} + \frac{1}{T_d} < \omega < \frac{1}{T_d}$$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{2\pi j (T_b/F)}{(C_l - C_m)} \cdot \left\{ \frac{1}{(C_i - C_m)} f(C_i \omega) f((C_m - C_l) \left(\frac{2}{T_c}\right)) \cdot f((C_m - C_l) \left(\frac{1}{T_d}\right)) \right. \\ \left. + \frac{1}{(C_i + C_m - 2C_l)} f((2C_l - C_i) \omega) f((C_m - C_l) \left(\frac{2}{T_c}\right)) f((C_i + C_m - 2C_l) \left(\frac{1}{T_d}\right)) \right\}$$

(4) For  $\frac{-2}{T_c} - \frac{1}{T_d} < \omega < \frac{-2}{T_c} + \frac{1}{T_d}$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{2\pi j (T_b/F)}{(C_m - C_l)} \frac{1}{(C_i - C_m)} f((2C_m - C_i) \omega) f((2C_m - C_l - C_i) \left(\frac{2}{T_c}\right)) f((C_i - C_m) \left(\frac{1}{T_d}\right))$$

(5) For  $\omega < \frac{-2}{T_c} - \frac{1}{T_d} \rightarrow X_{o_1}(\omega, z) = 0$

When the pulse-width of the data is smaller than the pulse-width of clock ( $T_d < T_c$ ) we have:

(1) For  $\omega > \frac{2}{T_c} + \frac{1}{T_d} \rightarrow X_{o_1}(\omega, z) = 0$

(2) For  $\frac{1}{T_d} < \omega < \frac{2}{T_c} + \frac{1}{T_d}$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{1}{(C_l - C_m)} \cdot \frac{2\pi j (T_b/F)}{(C_i + C_m - 2C_l)} f((C_l - C_i) \frac{2}{T_c}) f((C_l - C_m - 2C_l) \left(\frac{1}{T_d}\right)) f(C_i(\omega))$$

(3) For  $\frac{2}{T_c} - \frac{1}{T_d} < \omega < \frac{1}{T_d}$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{1}{(C_l - C_m)} \left\{ \frac{2\pi j (T_b/F)}{(C_i - C_m)} f(C_i(\omega)) f((C_m - C_l) \left(\frac{2}{T_c}\right)) f((C_i - C_m) \left(\frac{1}{T_d}\right)) \right. \\ \left. + \frac{1}{(C_i + C_m - 2C_l)} f((C_m - 2C_l)(\omega)) f((C_l - C_i) \left(\frac{2}{T_c}\right)) \right\}$$

(4) For  $\frac{-2}{T_c} + \frac{1}{T_d} < \omega < \frac{2}{T_c} - \frac{1}{T_d}$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{2\pi j (T_b/F)}{(C_l - C_m)} \cdot \left\{ \frac{1}{(C_i - C_m)} f(C_i(\omega)) f((C_m - C_l) \left(\frac{2}{T_c}\right)) f((C_i - C_m) \left(\frac{1}{T_d}\right)) \right. \\ \left. + \frac{1}{(C_i + C_m - 2C_l)} f((2C_l - C_i)(\omega)) f((C_m - C_l) \left(\frac{2}{T_c}\right)) f((C_i + C_m - 2C_l) \left(\frac{1}{T_d}\right)) \right\}$$

(5) For  $-\frac{1}{T_d} < \omega < \frac{-2}{T_c} + \frac{1}{T_d}$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{1}{(C_l - C_m)} \cdot \left\{ \frac{1}{(C_i - C_m)} f(C_m(\omega)) f((C_l + C_i - 2C_m) \left(\frac{2}{T_c}\right)) \right. \\ \left. + \frac{2\pi j \left(\frac{T_b}{F}\right)}{(C_i + C_m - 2C_l)} f((2C_l - C_i)(\omega)) f((C_m - C_i) \left(\frac{2}{T_c}\right)) f((C_i + C_m - 2C_l) \left(\frac{1}{T_d}\right)) \right\}$$

(6) For  $\frac{-2}{T_c} - \frac{1}{T_d} < \omega < -\frac{1}{T_d}$

$$X_{o_1}(\omega, z) = E_1 \sum_{m=1}^K \sum_{i=1}^K \sum_{l=1}^K \frac{2\pi j \left(\frac{T_b}{F}\right)}{(C_l - C_m)} \frac{1}{(C_i - C_m)} f((2C_m - C_i)(\omega)) f((2C_m - C_l - C_i) \left(\frac{2}{T_c}\right)) f((C_m - C_i) \left(\frac{1}{T_d}\right))$$

(7) For  $\omega < \frac{-2}{T_c} - \frac{1}{T_d}$

$$X_{o_1}(\omega, z) = 0$$

For calculating the  $X_{o_2}(\omega, z)$  and by defining  $S, S_1, S_2, S_3, S_4$  and  $E_2$  as:

$$E_2 = 2\pi^5 A_c'^3 T_c^3 b^2 A_d^2 T_d^2 \gamma_{osc} (\gamma_{ss} + 2\gamma_{os}) e^{G_1(\omega)z} \left( \frac{1 - e^{-2\alpha z}}{2\alpha} \right)^2$$

$$,S = \frac{1}{2\pi j (C_m - C_l) \cdot \left(\frac{T_b}{F}\right)}$$

$$,S_1 = \frac{1}{-2\pi j (C_p - C_m) \cdot \left(\frac{T_b}{F}\right)}$$

$$,S_2 = \frac{1}{-2\pi j (C_p + C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right)}$$

$$,S_3 = \frac{1}{2\pi j (C_n + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right)}$$

$$,S_4 = \frac{1}{2\pi j (C_n - C_p) \cdot \left(\frac{T_b}{F}\right)}$$

Then, for  $T_d > T_c$ ,  $X_{o_2}$  would be:

(1)  $\omega > \frac{3}{T_c} + \frac{2}{T_d}$

$$X_{o_2}(\omega, z) = 0$$

(2)  $\frac{3}{T_c} < \omega < \frac{3}{T_c} + \frac{2}{T_d}$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j (3C_i - 2C_l - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j (2C_i - C_n + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$. e^{-2\pi j C_i \cdot (T_b/F) \cdot \omega}$$

$$(3) \quad \frac{1}{T_c} + \frac{2}{T_d} < \omega < \frac{3}{T_c}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k S \cdot S_1 \cdot S_3 \cdot \left\{ \frac{1}{2\pi j (C_i + C_p - 2C_m) \cdot (T_b/F)} e^{2\pi j (3C_i - 2C_l - C_m) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (C_i + C_n + 2C_p - 2C_m) \cdot (T_b/F) \cdot (\frac{2}{T_d})} \right. \\ \left. e^{-2\pi j C_i \cdot (T_b/F) \cdot \omega} + \frac{1}{2\pi j (C_i - C_n) \cdot (T_b/F)} e^{2\pi j (-3C_i + 6C_n - 2C_l - C_m) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i - C_n + C_p - 2C_m) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j (C_i - 2C_n) \cdot (T_b/F) \cdot \omega} \right.$$

$$(4) \quad \frac{3}{T_c} + \frac{2}{T_d} < \omega < \frac{1}{T_c} + \frac{2}{T_d}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k S \cdot S_2 \cdot S_4 \cdot \frac{1}{2\pi j (C_i - C_n) \cdot (T_b/F)} e^{2\pi j (C_i - C_m) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i - C_n + C_p) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j C_i \cdot (T_b/F) \cdot \omega}$$

$$+ S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_i - C_n) \cdot (T_b/F)} e^{2\pi j (C_i + C_m - 2C_l) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i + C_p - C_n - 2C_m) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j (C_i) \cdot (T_b/F) \cdot \omega}$$

$$S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_i - C_n + 2C_p - 4C_m) \cdot (T_b/F)} e^{2\pi j (C_i + 3C_m - 2C_l - 2C_p) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i + 5C_p + 3C_n - 10C_m) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j (C_i) \cdot (T_b/F) \cdot \omega}$$

$$+ S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_n + C_p - 2C_m) \cdot (T_b/F)} e^{2\pi j (C_m - C_p + 2C_i - 2C_l) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (C_n - C_i) \cdot (T_b/F) \cdot (\frac{2}{T_d})} \cdot e^{-2\pi j (C_p - 2C_m) \cdot (T_b/F) \cdot \omega}$$

$$+ S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_i - C_n) \cdot (T_b/F)} e^{2\pi j (6C_n - 2C_l - C_m - 3C_i) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i - C_n + C_p - 2C_m) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j (C_i - 2C_n) \cdot (T_b/F) \cdot \omega}$$

$$(5) \quad \frac{1}{T_c} < \omega < \frac{3}{T_c} + \frac{2}{T_d}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k S \cdot S_2 \cdot S_4 \cdot \frac{1}{2\pi j (C_i - C_n) \cdot (T_b/F)} e^{2\pi j (C_i - C_m) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i - C_n + C_p) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j C_i \cdot (T_b/F) \cdot \omega}$$

$$+ S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_i - C_n) \cdot (T_b/F)} e^{2\pi j (C_i + C_m - 2C_l) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i + C_p - C_n - 2C_m) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j (C_i) \cdot (T_b/F) \cdot \omega}$$

$$S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_i - C_n + 2C_p - 4C_m) \cdot (T_b/F)} e^{2\pi j (C_i + 3C_m - 2C_l - 2C_p) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (2C_i + 5C_p + 3C_n - 10C_m) \cdot (T_b/F) \cdot (\frac{1}{T_d})} \cdot e^{-2\pi j (C_i) \cdot (T_b/F) \cdot \omega}$$

$$+ S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j (C_n + C_p - 2C_m) \cdot (T_b/F)} e^{2\pi j (7C_m - 4C_p - 2C_l - C_i) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (C_n + C_p - 2C_m) \cdot (T_b/F) \cdot (\frac{2}{T_d})} \cdot e^{-2\pi j (2C_p - C_i - 4C_m) \cdot (T_b/F) \cdot \omega}$$

$$(6) \quad \frac{-1}{T_c} + \frac{2}{T_d} < \omega < \frac{1}{T_c}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k S \cdot S_2 \cdot S_4 \cdot \frac{1}{2\pi j (C_i - C_p) \cdot (T_b/F)} e^{2\pi j (C_i - C_m) \cdot (T_b/F) \cdot (\frac{1}{T_c})} \cdot e^{2\pi j (C_n - C_p) \cdot (T_b/F) \cdot (\frac{2}{T_d})} \cdot e^{-2\pi j C_i \cdot (T_b/F) \cdot \omega}$$

$$\begin{aligned}
& +S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j(C_i + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_m - 2C_l + C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{2}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
& +S \cdot S_2 \cdot S_4 \cdot \frac{1}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_i - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_p - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
& +S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_n - C_i + C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + C_p - C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
& S \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(9C_m - C_i - C_n - 2C_l - 5C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 5C_p + 3C_n - 10C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 4C_p + 2C_n - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}
\end{aligned}$$

$$(7) \quad \frac{1}{T_c} - \frac{2}{T_d} < \omega < \frac{-1}{T_c} + \frac{2}{T_d}$$

$$\begin{aligned}
& X_{O_2}(\omega, z) \\
& = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \left\{ \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_l - 3C_p - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \right. \\
& \quad + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_n + C_i - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(2C_p - C_m - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n - 5C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} + \\
& \quad \frac{S \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j(C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_p - 2C_l - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n + 5C_p - 10C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \\
& \quad + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j C_p \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(2C_i - C_m - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} + \\
& \quad \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j(2C_m - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(2C_i + C_p - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p + 2C_m - 2C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \\
& \quad + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(2C_n - C_m - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} + \\
& \quad \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(C_m + 2C_n - 2C_l - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + C_p - C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \\
& + \\
& \quad \frac{S \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(11C_m - 6C_p - 2C_l - 2C_n - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n + 5C_p - 10C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}
\end{aligned}$$

$$(8) \quad \frac{-3}{T_c} + \frac{2}{T_d} < \omega < \frac{-1}{T_c}$$

$$\begin{aligned}
& X_{O_2}(\omega, z) \\
& = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \left\{ \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_l - 3C_p - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \right. \\
& \quad + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_n + C_i - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(2C_p - C_m - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n - 5C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} + \\
& \quad \frac{S \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{-2\pi j(C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_p - 2C_l - C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n + 5C_p - 10C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}
\end{aligned}$$





$$(13) \quad \omega < \frac{-3}{T_c} - \frac{2}{T_d}$$

$$X_{o_2}(\omega, z) = 0$$

When the pulse-width of the data is smaller than the pulse-width of clock ( $T_d < T_c$ ) for  $X_{o_2}$  we have:

$$(1) \quad \omega > \frac{3}{T_c} + \frac{2}{T_d}$$

$$X_{o_2}(\omega, z) = 0$$

$$(2) \quad \frac{3}{T_c} < \omega < \frac{3}{T_c} + \frac{2}{T_d}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k S_1 \cdot S_1 \cdot S_3 \cdot \frac{1}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(3C_i - 2C_l - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(3) \quad \frac{3}{T_c} < \omega < \frac{1}{T_c} + \frac{2}{T_d}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S_1 \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_i - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_p - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_i + C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + C_p - C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S_1 \cdot S_3}{2\pi j(C_i + C_p - 2C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_n + C_i - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{2}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + C_n - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_p + C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(4) \quad -\frac{1}{T_c} + \frac{2}{T_d} < \omega < \frac{3}{T_c}$$

$$X_{o_2}(\omega, z) = E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S_1 \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(3C_i - 2C_n - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(3C_i - 2C_n + C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_p + C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(3C_i + 2C_n - 5C_m - 2C_l + 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_p + C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S_1 \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(3C_n - 2C_i - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_n \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$





$$\begin{aligned}
& \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m)} e^{2\pi j(15C_m - 2C_i - 2C_l - 8C_p - 3C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 5C_p + 3C_n - 10C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j(C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \omega} \\
& + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_i - C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \omega} \\
& + \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_i + C_m - 2C_l) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n + C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \omega} \\
& + \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_i - 2C_p + 3C_m - 2C_l) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n + C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}
\end{aligned}$$

$$(8) \quad \frac{-1}{T_c} < \omega < \frac{-3}{T_c} + \frac{2}{T_d}$$

$$X_{O_2}(\omega, z)$$

$$= E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j C_i \cdot \left(\frac{T^b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_l - 8C_p - 3C_i - 2C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 2C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_n + C_i - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j C_i \cdot \left(\frac{T^b}{F}\right) \cdot \omega} \cdot e^{2\pi j(6C_p - C_m - 3C_i - 2C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n - 5C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} +$$

$$\frac{S \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j(C_i) \cdot \left(\frac{T^b}{F}\right) \cdot \omega} \cdot e^{2\pi j(13C_m - 2C_n - 6C_p - 2C_l - 3C_i) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n + 5C_p - 10C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$\frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(4C_i - 5C_n + 5C_m - 2C_l - 3C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_n - 2C_i - C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_n \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(5C_n - C_m - 8C_p + 4C_l) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_p - 2C_i - C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{2}{T_d}\right)} \cdot e^{2\pi j(C_n - 2C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$\frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(4C_i + 5C_n - 15C_m - 2C_l + 8C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(6C_m - 3C_p - C_n - 2C_l) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$\frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(5C_m + C_n - 2C_i - 2C_l - 3C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n - C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_n \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i - C_p) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(2C_n + C_p - 2C_i - 2C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 2C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j(C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(3C_m + 2C_n - C_p - 2C_l - 2C_i) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{2}{T_d}\right)} \cdot e^{2\pi j(C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$\frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(15C_m - 2C_i - 2C_l - 8C_p - 3C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 5C_p + 3C_n - 10C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j(C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_i - C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_i + C_m - 2C_l) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n + C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_i - 2C_p + 3C_m - 2C_l) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n + C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T^b}{F}\right) \cdot \omega}$$

$$(9) \quad \frac{1}{T_c} - \frac{2}{T_d} < \omega < -\frac{1}{T_c}$$

$$\begin{aligned}
& X_{O_2}(\omega, z) \\
&= E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \left\{ \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j C_i \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(5C_m - 2C_l - 4C_p - C_i + 2C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \right. \\
&\quad \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i - C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j C_p \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(5C_m - 2C_l - 3C_p - 2C_i + 2C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_p - 2C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \\
&+ \frac{S_2 \cdot S_2 \cdot S_4}{2\pi j(C_n + C_i - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j(C_n - 2C_p) \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(4C_p - C_m - 2C_i - C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_p - 2C_i - C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} + \\
&\quad \frac{S_1 \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j(2C_p + C_n - 4C_m) \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(9C_m - C_n - 4C_p - 2C_l - 2C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(6C_m - 2C_i - C_n - 3C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \\
&\quad \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(4C_i - 5C_n + 5C_m - 2C_l - 3C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_n - 2C_i - C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_n \left(\frac{T^b}{F}\right) \omega} \\
&+ \frac{S_2 \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(5C_n - C_m - 8C_p + 4C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_p - 2C_i - C_n) \left(\frac{T^b}{F}\right) \left(\frac{2}{T_d}\right)} \cdot e^{2\pi j(C_n - 2C_p) \left(\frac{T^b}{F}\right) \omega} \\
&\quad \frac{S_1 \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(4C_i + 5C_n - 15C_m - 2C_l + 8C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(6C_m - 3C_p - C_n - 2C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_n + 2C_p - 4C_m) \left(\frac{T^b}{F}\right) \omega} \\
&\quad \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(5C_m - C_i - 2C_l - 3C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n - C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - C_n) \left(\frac{T^b}{F}\right) \omega} \\
&+ \frac{S_2 \cdot S_2 \cdot S_4}{2\pi j(C_i - C_p) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(2C_n - C_i - 2C_m) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 2C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_p) \left(\frac{T^b}{F}\right) \omega} \\
&\quad + \frac{S_1 \cdot S_1 \cdot S_3}{2\pi j(C_i + C_p - 2C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_m + 2C_n - 2C_l - C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 2C_p - 4C_m) \left(\frac{T^b}{F}\right) \left(\frac{2}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_p - 4C_m) \left(\frac{T^b}{F}\right) \omega} \\
&\quad \frac{S_1 \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(11C_m - C_i - 2C_l - 6C_p - 2C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 5C_p + 3C_n - 10C_m) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j(C_i + 2C_n + 4C_p - 8C_m) \left(\frac{T^b}{F}\right) \omega} \\
(10) \quad & \frac{-3}{T_c} < \omega < \frac{1}{T_c} - \frac{2}{T_d}
\end{aligned}$$

$$\begin{aligned}
& X_{O_2}(\omega, z) \\
&= E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \left\{ \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j C_i \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(5C_m - 2C_l - 4C_p - C_i + 2C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \right. \\
&\quad \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i - C_p) \cdot \left(\frac{T^b}{F}\right)} e^{-2\pi j C_p \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(5C_m - 2C_l - 3C_p - 2C_i + 2C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_p - 2C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \\
&+ \frac{S_2 \cdot S_2 \cdot S_4}{2\pi j(C_n + C_i - 2C_p) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(C_n - 2C_p) \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(4C_p - C_m - 2C_i - C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_p - 2C_i - C_n) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} + \\
&\quad \frac{S_1 \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(2C_p + C_n - 4C_m) \left(\frac{T^b}{F}\right) \omega} \cdot e^{2\pi j(9C_m - C_n - 4C_p - 2C_l - 2C_i) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(6C_m - 2C_i - C_n - 3C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \\
&\quad \frac{S_1 \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T^b}{F}\right)} e^{2\pi j(3C_i - 4C_n + 5C_m - 2C_l - 3C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \left(\frac{T^b}{F}\right) \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \left(\frac{T^b}{F}\right) \omega}
\end{aligned}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(4C_n - C_m - 2C_p + 3C_i) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{2}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n - 4C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$\frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(3C_i + 4C_n + 6C_p - 11C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_p + C_n - 2C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(11) \quad \frac{-1}{T_c} - \frac{2}{T_d} < \omega < \frac{-3}{T_c}$$

$X_{O_2}(\omega, z)$

$$= E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_l - 10C_p + 2C_i + 5C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$\frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_i - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_l - 6C_p + C_i + 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_n + C_i - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_i + 2C_n - 4C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(C_i - C_m - 2C_p + 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 5C_p + 3C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} +$$

$$\frac{S \cdot S_1 \cdot S_3}{2\pi j(2C_p + C_n + C_i - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(4C_p + 2C_n + C_i - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(2C_n + 2C_p - 2C_l + C_i - 4C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(3C_n + 2C_l - 10C_m + 5C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$(12) \quad \frac{-3}{T_c} - \frac{2}{T_d} < \omega < \frac{-3}{T_c}$$

$X_{O_2}(\omega, z)$

$$= E_2 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(C_i + 2C_n - 4C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \cdot e^{2\pi j(5C_m - 2C_l - 12C_p + 3C_i + 6C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 3C_n - 5C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)}$$

$$(13) \quad \omega < \frac{-3}{T_c} - \frac{2}{T_d}$$

$$X_{O_2}(\omega, z) = 0$$

For calculating the  $X_{O_3}(\omega, z)$  and by defining and  $E_3$  as:

$$E_3 = 8\pi^5 A_c'{}^4 T_c{}^4 b A_d T_d 8\gamma_{ORC}(\gamma_{RC} + 2\gamma_{OC}) e^{G_1(\omega)z} \left(\frac{1 - e^{-2\alpha z}}{2\alpha}\right)^2$$

Then, for  $T_d > T_c$ ,  $X_{O_2}$  would be:

$$(1) \quad \omega > \frac{4}{T_c} + \frac{1}{T_d}$$

$$X_{O_3}(\omega, z) = 0$$

$$(2) \quad \frac{2}{T_c} + \frac{1}{T_d} < \omega < \frac{4}{T_c} + \frac{1}{T_d}$$

$$X_{O_3}(\omega, z) = E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(4C_i + C_p - 2C_l - 3C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(3) \quad \frac{4}{T_c} - \frac{1}{T_d} < \omega < \frac{2}{T_c} + \frac{1}{T_d}$$

$$X_{O_3}(\omega, z) = E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i - C_p - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$







$$\begin{aligned}
X_{O_3}(\omega, z) &= E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(5C_m + 2C_n - 2C_l - 2C_i - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
&\quad + \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i - 2C_n + 5C_m - 2C_l - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
&\quad + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + 4C_n + 3C_p - 9C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
&\quad + \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + C_p - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
(9) \quad &\frac{-2}{T_c} - \frac{1}{T_d} < \omega < -\frac{4}{T_c} + \frac{1}{T_d}
\end{aligned}$$

$$\begin{aligned}
X_{O_3}(\omega, z) &= E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + 5C_m + 6C_n - 2C_l - 11C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
&\quad + \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i - 2C_n + 5C_m - 2C_l - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
&\quad + \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + 4C_n + 3C_p - 9C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
&\quad + \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + C_p - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
(10) \quad &\frac{-4}{T_c} - \frac{1}{T_d} < \omega < -\frac{2}{T_c} - \frac{1}{T_d}
\end{aligned}$$

$$\begin{aligned}
X_{O_3}(\omega, z) &= E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(4C_i + 5C_m + 8C_n - 2C_l - 15C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n - 4C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega} \\
(11) \quad &\omega < \frac{-4}{T_c} - \frac{1}{T_d}
\end{aligned}$$

$$X_{O_3}(\omega, z) = 0$$

When the pulse-width of the data is smaller than the pulse-width of clock ( $T_d < T_c$ ) for  $X_{O_3}$  we have:

$$(1) \quad \omega > \frac{4}{T_c} + \frac{1}{T_d}$$

$$X_{O_3}(\omega, z) = 0$$

$$(2) \quad \frac{4}{T_c} - \frac{1}{T_d} < \omega < \frac{4}{T_c} + \frac{1}{T_d}$$

$$X_{O_3}(\omega, z) = E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(4C_i + C_p - 2C_l - 3C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$



$$\frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_n + C_p - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(5C_m - 2C_l - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(7) \quad \frac{-2}{T_c} + \frac{1}{T_d} < \omega < -\frac{1}{T_d}$$

$$X_{o_3}(\omega, z) = E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(5C_m - 2C_l - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_n \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_n - C_p - C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_n + C_p - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_n \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(8) \quad \frac{-2}{T_c} - \frac{1}{T_d} < \omega < \frac{-2}{T_c} + \frac{1}{T_d}$$

$$X_{o_3}(\omega, z)$$

$$= E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(5C_m + 2C_n - 2C_l - 2C_i - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{-2\pi j C_i \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i - 2C_n + 5C_m - 2C_l - 3C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_2 \cdot S_4}{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + 4C_n + 3C_p - 9C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n + 2C_p - 4C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n + 4C_p - 8C_m) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$+ \frac{S \cdot S_1 \cdot S_3}{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(2C_i + C_p - C_m - 2C_l) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i - C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i - 2C_n) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(9) \quad \frac{-4}{T_c} + \frac{1}{T_d} < \omega < -\frac{2}{T_c} - \frac{1}{T_d}$$

$$X_{o_3}(\omega, z)$$

$$= E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(5C_m + 4C_n - 2C_l - 7C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(2C_i + 2C_n - 4C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(10) \quad \frac{-4}{T_c} - \frac{1}{T_d} < \omega < -\frac{4}{T_c} + \frac{1}{T_d}$$

$$X_{o_3}(\omega, z) = E_3 \sum_{m=1}^k \sum_{n=1}^k \sum_{p=1}^k \sum_{i=1}^k \sum_{l=1}^k \frac{S \cdot S_1 \cdot S_4}{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right)} e^{2\pi j(4C_i + 5C_m + 8C_n - 2C_l - 15C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_c}\right)} \cdot e^{2\pi j(C_i + C_n - 2C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \left(\frac{1}{T_d}\right)} \cdot e^{2\pi j(C_i + 2C_n - 4C_p) \cdot \left(\frac{T_b}{F}\right) \cdot \omega}$$

$$(11) \quad \omega < \frac{-4}{T_c} - \frac{1}{T_d}$$

$$X_{o_3}(\omega, z) = 0$$